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Surname	Other names

Pearson
Edexcel GCE

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C3

Advanced

Tuesday 19 June 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
6665/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Given $y = 2x(3x - 1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \leqslant 0$

(2)

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Question 1 continued

Q1

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2. The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \qquad x > 4$$

(a) Show that $f(x) = \dfrac{A}{Bx+C}$ where A, B and C are constants to be found.

(4)

(b) Find $f^{-1}(x)$ and state its domain.

(3)

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Question 2 continued

Q2

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3. The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \qquad t \geqslant 0, t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(a) find the value of A,

(1)

(b) show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

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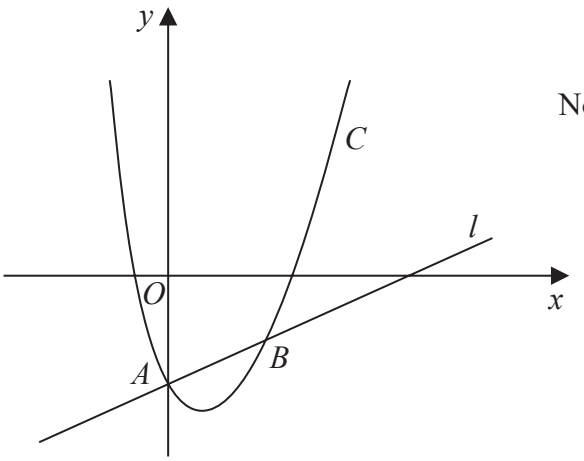
Question 3 continued

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Q3

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4.



Not to scale

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B, as shown in Figure 1.

(b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find x_2 and x_3 to 3 decimal places.

(2)

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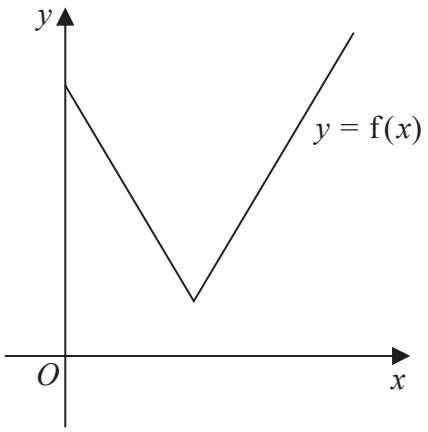
Question 4 continued

Q4

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5.



Not to scale

Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|5 - x| + 3, \qquad x \geqslant 0$$

Given that the equation $f(x) = k$, where k is a constant, has exactly one root,

(a) state the set of possible values of k.

(2)

(b) Solve the equation $f(x) = \frac{1}{2}x + 10$

(4)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = 4f(x - 1)$. The vertex on the graph with equation $y = 4f(x - 1)$ has coordinates (p, q).

(c) State the value of p and the value of q.

(2)

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Question 5 continued

Q5

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6. (i) Using the identity for $\tan(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°, \ n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

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Question 6 continued

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Q6

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7. The curve C has equation $y = \dfrac{\ln(x^2 + 1)}{x^2 + 1}, \quad x \in \mathbb{R}$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a single fraction, simplifying your answer.

(3)

(b) Hence find the exact coordinates of the stationary points of C.

(6)

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Question 7 continued

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8. (a) By writing $\sec\theta = \dfrac{1}{\cos\theta}$, show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta) = \sec\theta\tan\theta$

(2)

(b) Given that

$$x = \mathrm{e}^{\sec y} \qquad x > \mathrm{e}, \qquad 0 < y < \frac{\pi}{2}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x\sqrt{\mathrm{g}(x)}}, \qquad x > \mathrm{e}$$

where $\mathrm{g}(x)$ is a function of $\ln x$.

(5)

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9. (a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $M(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $N(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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TOTAL FOR PAPER: 75 MARKS

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